Comment on "Unification of Matrix Methods of Structural Analysis"

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RECENTLY, Filho[1] presented a brief discussion attempting to illustrate the equivalence of certain methods of matrix structural analysis. Only one simple example was presented based on a very early paper of the present writer. It was claimed that the method being developed by the present author is synonymous with other methods.

The purpose of the present note is to state some important differences between this method and others. These differences have been shown already in work of the present author.[2, 3] They are as follows:

1) It is not necessary to use a lumped discrete element idealization in applying the method. The structure can be represented as it really is in nature. Orthotropic or aeolotropic plate and shell theory are useful in this respect.

2) Either a stiffness or a flexibility formulation can be used, or both can be used, as desired, in the same problem.

3) The equations are written in terms of definite integrals or derivatives of functions such that the results are very accurate.

4) Any kind of special equation can be written, such as continuity, boundary condition, etc. In other words, there is more freedom in the choice of equations and in the order of writing down the equations.

5) Both equilibrium and structural continuity are satisfied directly simultaneously.

Therefore, it is necessary to study the foregoing items before drawing any conclusions.

References

[1] Filho, F. V., "Unification of matrix methods of structural analysis," AIAA J. **1**, 916–917 (1963).

[2] Klein, B., "A simple method of matric structural analysis," J. Aerospace Sci., Part III **25**, 385–394, Part IV **26**, 351–359, Part V **27**, 859–866, Part VI **29**, 306–310 (1959–1961).

[3] Klein, B., "A simple method of matric structural analysis," Parts VII and VIII (to be published).

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Reply by Author to B. Klein

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THE main purpose of the writer in presenting the article commented upon by Klein was to demonstrate that "Argyris' equations are exactly Klein's after the ideal pre-triangularization is obtained."[1]

Regarding item 1 of Klein's comments, in the papers Klein has so far published only a lumped discrete element idealization is used. The writer is looking forward to the publication of parts VII and VIII of Klein's work to learn how "the structure can be represented as it really is in nature." It must be stated, however, that plates and shells have already been treated by matrix methods.[2]

Items 2–5 of Klein's comments appear to the writer to be features that are more or less present in any method of analysis of linear elastic structures. In fact, all these methods have their basis in the old principles of Maxwell, Mohr, and Mueller-Breslau. The proliferation of methods of structural analysis must lead us to search for what is fundamentally new in a method and not to be blinded with formal details.

References

[1] Filho, F. V., "Unification of matrix methods of structural analysis," AIAA J. **1**, 916–917 (1963).

[2] Hessel, A., "Analysis of plates and shells by matrix methods," Svenska Aeroplan Aktiebolaget TN 48 (1961).

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Correction to "Special Solutions to the Equations of Motion for Maneuvering Entry"

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IN Ref. 1, Jackson presented a very useful technique for analytical analysis of the lateral maneuver. His final Eq. (12) should be corrected, however. Combining his Eqs. (4) and (11) to eliminate the heading angle and to express the longitude θ in terms of the latitude ϕ for an equatorial entry ($Q = 1$) yields

$$d\theta = \frac{d\phi}{\cos\phi}\left(\frac{1 - \sin\phi}{2 \sin\phi}\right)^{1/2} \tag{1}$$

Integrating,

$$\sin\theta = \left(\frac{2 \sin\phi}{1 + \sin\phi}\right)^{1/2} \tag{2}$$

which replaces Eq. (12) of Ref. 1.

Reference

[1] Jackson, W. S., "Special solutions to the equations of motion for maneuvering entry," J. Aerospace Sci. **29**, 236 (1962).

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Comment on "Turbulent Mixing of Compressible Free Jets"

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IN a recent technical note,[1] Maydew and Reed give some of the results of an experimental investigation that they conducted.[2] They point out that considerable effort was devoted to trying to fit the data (by the *right choice* of σ, the jet-spreading parameter) to the error function distribution and also to Crane's (or Gortler's) incompressible solution with the result that Crane's profile gave the best fit. The purpose of this note is to mention a much easier method of determining σ than that of having to make various choices for σ and then compare the data with an appropriate theoretical solution.

For purposes of determining the virtual origin of the mixing region, Maydew and Reed (Ref. 2, p. 21) plotted the mixing region width $b_{0.1}$ vs x, the axial distance. The width $b_{0.1}$ is by definition the radial distance between the points where $(V/V_1)^2$ is 0.1 and 0.9. The data were given as

$$b_{0.1} = C(x + a)$$

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where C is a spread constant, and a is the distance to the virtual origin. For a given theoretical distribution (e.g., Crane's theory), the value of $\xi[= \sigma y/(x + a)]$ corresponding to $b_{0.1}$ may be found easily $[\xi_{0.1} = 1.02 - (-0.37) = 1.39]$. The value of σ then may be computed from a knowledge of C by

$$\sigma = \frac{x+a}{y}\,\xi = \frac{x+a}{b_{0.1}}\,\xi_{0.1} = \frac{\xi_{0.1}}{C} = \frac{1.39}{C}$$

for Crane's profile. For the error function profile, 1.39 would be replaced by 1.49, thus showing, as pointed out by Maydew and Reed, that the value of σ is somewhat dependent upon the choice of profile. Thus, having calculated σ in the forementioned way, one can make a comparison between various theoretical distributions without having to resort to the choices for σ.

Table 1 shows the values of σ found by Maydew and Reed as compared to values calculated from $1.39/C$ with C taken from Ref. 2, p. 21.

Table 1 Comparison of spread parameter values

M	Maydew and Reed	$1.39/C$
0.70	10.5	10.7
0.85	10.8	10.9
0.95	11.0	11.0
1.49	15.0	16.0
1.96	20.0	20.4

As is expected, the agreement is quite good.

References

[1] Maydew, R. C. and Reed, J. F., "Turbulent mixing of compressible free jets," AIAA J. 1, 1443–1444 (1963).

[2] Maydew, R. C. and Reed, J. F., "Turbulent mixing of axisymmetric compressible jets (in the half-jet region) with quiescent air," Sandia Corp. Res. Rept. SC-4764 (March 1963).

Comments on Aerodynamic Plane Change

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Nomenclature

A = reference area
C_D = drag coefficient
D = aerodynamic drag
L = aerodynamic lift (resultant of vertical and lateral aerodynamic forces)
V = velocity
W = weight
γ = roll angle measured from local vertical
Δi = change of inclination of trajectory plane
θ = flight path angle, relative to local horizontal

Subscript

E = conditions at atmospheric entry

IT has become apparent from digital computer studies that the approximate analysis of an aerodynamic plane change maneuver which was the subject of an earlier paper[1] by this writer is of much more limited validity than was indicated therein. The primary limitation of that analysis arises from the assumption that the vertical component of aerodynamic lift is much greater than the difference between centrifugal force and gravity. This assumption, which results in solutions describing a skip out of the atmosphere, leads to serious inaccuracies if (L/D) cosγ < about 1.0, since for entries at circular or subcircular speeds a skip may not even occur because of inadequate vertical lift. For reasonable L/D's, this therefore restricts the validity of the analysis of Ref. 1 to relatively small changes in inclination. For an L/D of 2.0, for example, this would restrict the roll angle to about 60° or less and the resulting Δi to about $2(3)^{1/2}\theta_E$ [see Eq. (13) of Ref. 1].

The consequences of these restrictions are that for large Δi (Ref. 1 included results for Δi up to 90°) the approximate analysis incorrectly describes the details of the vertical motion, i.e., the velocity, flight path angle, acceleration, and heating histories; in addition, the change in inclination predicted as a function of roll angle and entry angle is incorrect.

On the other hand, it is also clear on the basis of both intuition and computational results that the simplifying assumptions in the equations of tangential and lateral acceleration (see Ref. 1) are quite valid for practically any case of interest. For small flight path angles ($\theta^2 \ll 1$), these two equations can be combined and integrated to give

$$\Delta i = (L/D)\sin\gamma\,\ln(V_E/V) \tag{1}$$

where it has been assumed that (L/D) sinγ is constant during the maneuver. This equation, with γ = 90°, was the basis of Fig. 1 of Ref. 1. Limited computational results, shown in Fig. 1 of this note, indicate that this equation predicts Δi with a high degree of accuracy even for large Δi, and that, in fact, it does so conservatively, i.e., for given (L/D) sinγ (ratio of lateral force to drag) and velocity ratio, Δi will actually be slightly greater than that predicted by Eq. (1).

The significance of this result lies in the fact that Eq. (1) is independent of the details of the motion in the vertical

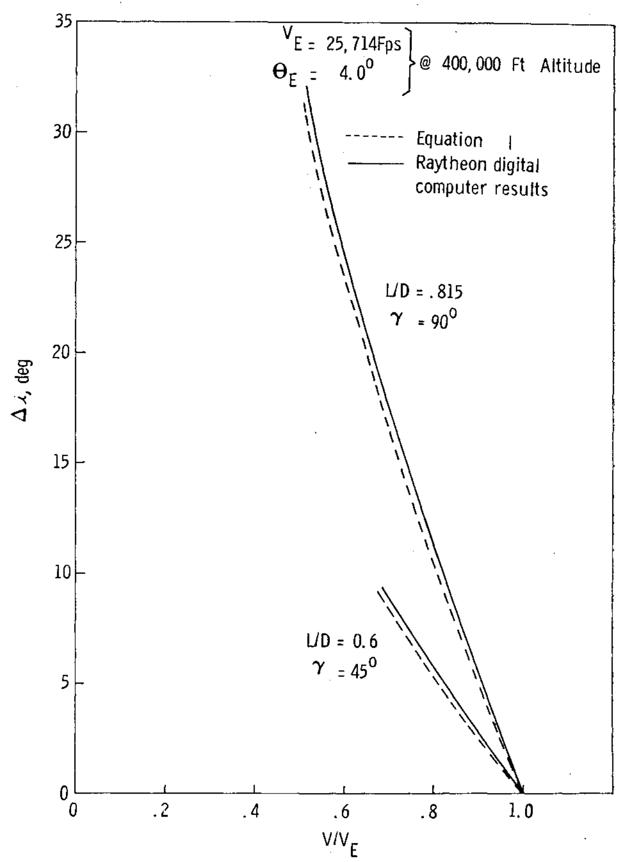

Fig. 1 Approximate analytical results from Eq. (1) as compared with numerical integration of equations of motion on a digital computer.

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